Economic Sustainment in Attritional Warfare: A Case Study of the Russian-Ukrainian Conflict

Ekonomická udržitelnost v opotřebovávací válce: případová studie rusko-ukrajinského konfliktu

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Abstract: This article presents a formal economic framework for assessing the sustainability of prolonged attritional warfare. Using the ongoing Russian invasion of Ukraine as a case study, the paper models the interaction between military production and consumption (losses) to evaluate the limits of state endurance under sustained conflict. The core of the analysis is a differential function of the war economy, defined as the net capacity to sustain operations over time, which is the difference between production and losses. This function is embedded within a theoretical construct that distinguishes between sustainable and unsustainable operational regimes, marked by critical thresholds of system exhaustion. Through a combination of regression analysis and symbolic modelling, the paper demonstrates how Russia's current attritional strategy may be approaching a structural breaking point. The findings offer a conceptual and empirical basis for evaluating wartime industrial resilience and may inform strategic policy and conflict forecasting.

Abstrakt: Tento příspěvek představuje formální ekonomický rámec pro hodnocení udržitelnosti při dlouhodobém opotřebovávacím válčení. Na příkladu probíhající ruské invaze na Ukrajinu modeluje interakci mezi vojenskou výrobou a spotřebou (ztrátami), aby zhodnotil meze odolnosti státu v podmínkách trvalého konfliktu. Jádrem analýzy je diferenciální funkce válečné ekonomiky, definovaná jako čistá schopnost udržet operace v čase, která představuje rozdíl mezi produkcí a ztrátami. Tato funkce je zakotvena v teoretické konstrukci, která rozlišuje mezi udržitelnými a neudržitelnými operačními režimy a je charakterizována kritickými mezemi vyčerpání systému. Na základě kombinace regresní analýzy a symbolického modelování příspěvek ukazuje, jak se současná ruská strategie vyčerpávání může blížit bodu strukturálního zlomu. Zjištění nabízejí koncepční

a empirický základ pro hodnocení odolnosti válečného průmyslu a mohou být podkladem pro strategickou politiku a předvídání konfliktů.

Keywords: Attritional Warfare; Defence Economics; Regression Analysis; Strategic Sustainment; War Economy.

Klíčová slova: opotřebovávací válka; ekonomika obrany; regresní analýza; strategická udržitelnost; válečná ekonomika.

INTRODUCTION

Despite being undesired and unwilling, attritional warfare, under certain circumstances, may become a strategic choice. In the case of Ukraine, assessing the enemy's resolve, resources, and capacity to replenish losses becomes essential for predicting the war's course. Kyrylo Budanov, head of Ukraine's Directorate of Intelligence, in one interview during the 2024 Annual Yalta European Strategy meeting, suggested that Ukraine could enter the final stages of its conflict with Russia, potentially concluding between late 2025 and early 2026, assuming that Russia is on the road to exhausting its resources for waging war (Тищенко 2024).

This study examines how long a state can sustain war efforts under the strain of attritional warfare. Using the Russian-Ukrainian conflict as a case study, it seeks to estimate the point at which military production can no longer meet losses, signalling operational collapse. The aim is to provide a practical, data-driven model for assessing economic sustainability in prolonged conflicts, aiding strategic decision-making where manoeuvre warfare is not feasible.

The concept of economic endurance developed in this paper has significant implications for conflict behaviour. As states approach their endurance threshold (marked as θ), the range of strategic options narrows. Actors facing systemic unsustainability are more likely to modify their strategic preferences, not necessarily because of a change in political will, but due to material constraints. This aligns with theories of *costly signalling* and *coercive bargaining*, where battlefield attrition alters perceived resolve and bargaining power (Fearon 1995).

This research intends to contribute to the body of knowledge on defence economics by providing econometric analysis to support strategic decision-making. It highlights the value of economic tools and resources over a doctrinal (maneuverist) perspective in specific circumstances, such as the war of attrition, a phase in which the Russia-Ukraine conflict has been, practically since its inception (i.e., February 2022).

1   METHODOLOGY

1.1   Aim, Methodology, Purpose and Scope of the Research

This research aims to estimate the duration for which the Russian military can sustain effective operations (offensive and defensive) given current trends in the consumption of military resources (i.e. the trends during the two years of the armed conflict between 2022 and 2024).

To achieve that aim, we established three objectives.
- The first objective **(O$_1$)** is to develop a detailed theoretical and graphical representation of the war economy function. In this manuscript, the war economy function represents the core mathematical construct developed to capture how a state's military capacity evolves during prolonged attritional warfare. The research is based on the premise that, in the case of Russia, the war economy, including the defence industrial base, provides the resources necessary to wage war efforts and sustain military (operational) activities, including defence industry production, refurbishment of stocked equipment, maintenance, and foreign aid.
- The second objective **(O$_2$)** is to determine when the Russian war economy reaches "the breaking point", i.e. exceeds the system's endurance threshold. This signifies the threshold at which the war economy can no longer sustain the required production--to-consumption ratio, directly affecting military and combat power. The research also aims to identify factors influencing the rate of change in this ratio.
- Finally, the third objective (**O$_3$**) is to use available data to assess the current state of the Russian war economy and to predict how long it can sustain its war efforts before a breakdown occurs.

The research aims to provide a theoretical model to support decision-makers in evaluating the strategic viability of attritional warfare, especially when manoeuvre-based strategies are impractical.

In terms of scope, our attrition warfare model is focused on analysing (primarily) Russian capabilities in terms of resources and production against losses between 2022 and 2024, with a 2026 horizon. While omitting the solid part of the DOTMLPF-I[1] spectrum in capabilities (NATO 2016), our study aims to encourage strategic planners (strategists) to recognise that attritional warfare, although undesirable, may be viewed as an

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[1]   DOTMLPF-I spectrum consists of eight components: Doctrine, Organisation, Training, Materiel, Leadership, Personnel, Facilities, and Interoperability. This framework outlines the fundamental elements necessary for effective capability development.

opportunity under specific circumstances. The scope of our research was intentionally limited to one operational domain, land, and a few capability areas, specifically "engage" and "sustainment" (NATO BiSC 2016), to simplify our approach. The complexity of waging warfare, developing strategies and operational plans, and ensuring sustainment (including the defence industrial base and logistics) is significantly higher. This has to be considered, potentially in parallel with scenario analysis.

1.2   Theoretical Framework

The research combines mathematical modelling and econometric techniques to analyse the dynamics of a war economy in an attritional style of conflict. From theoretical and methodological perspectives, the study builds upon a broad volume of literature. Regarding defence economics, the main framework for analysing war economies draws from Todd Sandler's and Keith Hartley's Handbooks on Economics of Defence, Volume 1 (Hartley and Sandler 1995) and Volume 2 (Sandler and Hartley, 2007). Elaborations on defence economics also include Ron Smith's studies on military production and economic sustainability (Smith 2009; Smith 2014). Regarding the economic modelling of demand and supply, the production-to-consumption ratio analysis, our research attempts to align with classical economic equilibrium theories, particularly those found in works such as Paul Samuelson's "Foundations of Economic Analysis" (Samuelson 1983) and Gerard Debreu's "Theory of Value" (Debreu 1972), The use of elasticity to determine the sustainability of the war economy is rooted in economic dynamics and calculus-based optimisation models, as detailed in works like Chiang and Wainwright's "Fundamental Methods of Mathematical Economics" (Chiang and Wainwright 2005). The graphical models and logarithmic trend equations align with established methodologies in mathematical economic modelling, as detailed in works like Ekeland and Temam's "Convex Analysis and Variational Problems" (Ekeland and Temam 1987).

Regarding econometric techniques, the trendlines and regression models used to analyse production and consumption rates, we used standard econometric tools such as those developed by William Greene in "Econometric Analysis" (Greene 2003) and Jeffrey Wooldridge in "Introductory Econometrics" (Wooldridge 2002). The methodology examining the trajectory of production and losses over quarters leverages principles of time-series analysis, as deliberated in "Time Series Analysis" by James D. Hamilton (Hamilton 1994), while using data to validate trends in the war economy empirically relates to the empirical analysis frameworks outlined in "Mostly Harmless Econometrics" by Angrist and Pischke (Angrist and Pischke 2009).

Conceptualising attritional warfare models stems from concepts of attritional warfare and its implications for resources and economic dynamics, which can be found in historical and strategic analyses, such as Trevor Dupuy's "Attrition: Forecasting Battle Casualties and Equipment Losses in Modern War" (Dupuy 1990). Manoeuvre as a way of waging warfare has traditionally been presented as a superior approach to attrition,

characterised as incremental, costly, and time-consuming. NATO's doctrine for Land Operations AJP 3.2 favours the "manoeuvrist approach" as "the land component's operational philosophy" (NSO 2022 37). The U.S. Marine Corps's Warfighting concept considers manoeuvres as a "state of mind" and "philosophy for 'fighting smart'" (USMC 2018, 4-26).

Waging quick and efficient military operations to achieve political objectives is *a de facto standard* in interstate armed conflicts. The two sides of the Russia-Ukraine war are no different. However, neither Russia nor Ukraine achieved its objectives quickly in 2022, nor did Ukraine succeed in a counteroffensive in the summer of 2023. Since then, the war in Ukraine has evolved into a war of attrition.

Military operations in an attritional conflict differ significantly from those in a war of manoeuvre. Rather than aiming for a decisive victory through swift and strategic movement, attritional warfare prioritises the gradual destruction of the enemy's forces and their capacity to replenish combat power while safeguarding one's resources (Vershinin 2024). A war of attrition may be characterised as a sustained effort where warring sides seek to progressively deplete each other's resources (including manpower) and resolve (the will to fight). Attritional warfare aims to impose relentless losses on the enemy over an extended period, ultimately rendering the opponent unable to continue the fight. Over time, exhaustion or resource depletion compels one side to retreat or surrender (Dupuy 1987).

2   RESULTS

2.1   Economic Modelling of the Dynamics of Attritional Warfare

To model the material dynamics of attritional warfare, we begin by formalising the structure of available military resources at the onset of sustained combat. This requires disaggregating the total pool of usable weaponry into specific components that reflect their operational status and origin, providing a clearer picture of how these resources are mobilised, deployed, and replenished over time. The total quantity of weaponry available for engagement at any given time $Q_{RT}(t)$ can be defined as:

$$Q_{RT}(t) = Q_{RO}(t) + Q_{RF}(t) + Q_{RP}(t) + Q_{RS}(t) + Q_{RM}(t) + Q_{RD}(t)$$

where:
R denotes Russia
Q_{RT} = Total quantity of weaponry available for engagement
Q_{RO} = Quantity of weaponry in operational use (immediately deployable)

Q_{RF} = Quantity of weaponry from foreign aid or imports
Q_{RP} = Quantity of newly produced weaponry ready for use
Q_{RS} = Quantity of weaponry in strategic reserves or stocks (including older versions)
Q_{RM} = Quantity of weaponry under maintenance or refurbishment (expected to return to service)
Q_{RD} = Quantity of assets in transit or awaiting deployment (e.g., in logistics pipeline)

This modular approach enables the monitoring of how Russia's weapon stock evolves, not only through production but also through the mobilisation of reserves**,** maintenance throughput, and foreign resupply.

For modelling purposes, we define Q_{RN}(t) as the nominal pool of potentially available weaponry not currently in operational use, but expected to become available through mobilisation, repair, or deployment: Q_{RN}(t) = Q_{RS}(t) $_+$ Q_{RM}(t) $_+$ Q_{RD}(t), representing the pool of potentially available weaponry that is not yet in active use but could be deployed.

To reflect actual usable capacity, we subtract losses over time:

$$Q_{RA}(t) = Q_{RT}(t) - Q_{RL}(t)$$

where Q_{RA} denotes the actual available force, and Q_R represents cumulative battle-field losses, obsolescence, or degradation.

After the onset of conflict, Q_{RT} naturally trends downward due to sustained losses. Only Q_{RP} (domestic production) and Q_{RF} (foreign imports) can grow, making them vital for long-term sustainment. Meanwhile, Q_{RM} (maintenance and repair) acts as a shock absorber, slowing the rate of decay but not reversing it. Also, Q_{RD}(t) has a logistics delay factor before becoming operational, while Q_{RM}(t) → Q_{RO} (t + Δt) over time, assuming successful repairs.
Point B marks the onset of a resource-driven phase of warfare, where initial stockpiles and capacities give way to sustained drawdowns and must be backed by new production or external inputs.
So:
- At Point B, Q_{RT}(t) is at its maximum practical level**.**
- After Point B, Q_{RL}(t) (losses) begin to rise.
- Unless replenished via Q_{RP}, Q_{RF} or Q_{RM} the system begins degrading

At Point B, the war economy enters a phase where the total available military arsenal (Q_{RT}) becomes a critical measure of strategic viability. This point represents the transition from pre-war accumulation and mobilisation to sustained resource depletion through attritional warfare. The available stock at B includes operational systems, reserves, imported arms, and those under maintenance or in deployment channels. Point B represents the threshold where most pre-war military assets, including reserves, imports, and domestic production, have been integrated into the system and are either operational

or positioned for deployment. Thus, we express total weapon availability at this baseline as:

$$Q^B_{RT}(t) = Q^B_{RO}(t) + Q^B_{RS}(t) + Q^B_{RM}(t) + Q^B_{RD}(t)$$

It has to be emphasised that $Q^B_{RF}(t)$ and $Q^B_{RP}(t)$ are not distinct sources at Point B because they have already been allocated to one of the above four categories. In addition, weapons received through foreign aid (Q_{RF}) and those from pre-war production (Q_{RP}) are assumed to have been absorbed into either operational use or reserve stockpiles. While we include Q_{RF} in the structural model, we do not incorporate it empirically in this analysis due to the uncertainty and inconsistency of available data on foreign military aid. See, for example, arms and ammunition transfers from North Korea (Balmforth and Zafra 2025; Bermudez et al. 2024). This omission does not affect the internal logic of the model, as Q_{RF} could be treated as an additive component to Q_{RP} (new production). However, in real-world applications, incorporating accurate and timely data on foreign--supplied weaponry would provide a more precise picture of sustainment capacity and trend dynamics, especially in conflicts where external support plays a substantial role.

In our model, from point B onward, Russia must rely on new flows, i.e., $Q_{RP}(t)$, $Q_{RF}(t)$ and recycled stock via $Q_{RM}(t)$, to offset attrition. Sustained mismatch between losses ($Q_{RL}(t)$) and replenishment will accelerate degradation beyond the baseline level. This baseline state reflects the peak aggregate of usable military assets. From this point onward, attrition begins to erode the available quantity ($Q_{RA}(t) = Q_{RT}(t) - Q_{RL}(t)$). Sustainability increasingly depends on the flow capacity of $Q_{RP}(t)$, $Q_{RF}(t)$ and $Q_{RM}(t)$. These are the only mechanisms capable of replenishing battlefield losses after B.

While our model focuses on the quantification of military resources, it is essential to emphasise that warfare, particularly in its planning and conduct, cannot be reduced to a mere comparison of quantities. Operational effectiveness, tactical innovation, leadership, morale, geography, and chance all play decisive roles that are not easily captured in theoretical frameworks. Nonetheless, the availability and sustainability of military resources constitute a foundational constraint on a state's ability to wage and endure prolonged conflict. Our approach does not attempt to simulate the full complexity of warfare; rather, it provides a conceptual and analytical model for understanding the dynamics of attritional warfare, where the balance between production, losses, and replenishment becomes a critical determinant of endurance over time. By isolating the resource dimension, we aim to illuminate how material constraints interact with strategic horizons, even in wars shaped by factors far more complex than material constraints alone.

In the context of warfare, the economics of attrition can be understood as the imbalance over time between the rate at which military capabilities are consumed (C), through combat losses and wear, and the rate at which they are replenished through production and supply (P). It is, therefore, the net difference between what a state can produce and what it loses in military resources over time.

Maintaining an adequate supply (P ≥ C), the warring side is better positioned to prevail in an attritional type of conflict. Figure 1 presents a graphical representation of this relationship, where the x-axis represents time (*t*), measured in quarters (Q), while the y-axis represents Net Sustainment Capacity (NSC), defined as the difference between military production and consumption.

$$NSC(Q)=P(Q)-C(Q)$$

By sustainment, we understand "*the provision of logistics and personnel services required to maintain and prolong operations until successful mission accomplishment*" (Defense Acquisition University 2024). We use the term "sustainment" whenever possible to distinguish it from "sustainability", which is used extensively in concepts such as sustainable development.
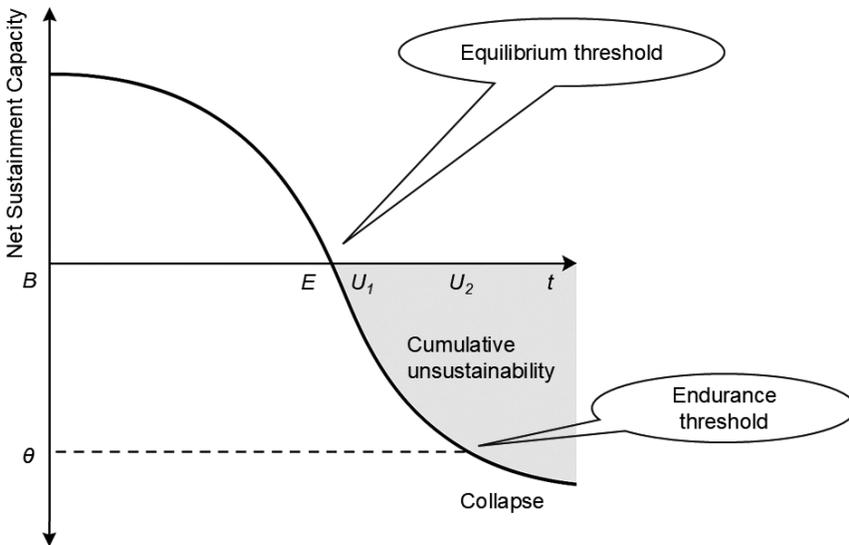
Figure 1: Graphical representation of the War Economy Function in attritional warfare

The two thresholds are *E* and *ϑ*. *E* denotes the point where production (still) equals consumption, and the war efforts are still sustained. The other is *ϑ*, which denotes the capacity under which the system reaches the "breaking point" (e.g. collapse) at time U_2, or the moment when the cumulative unsustainability D_U exceeds the critical threshold θ (theta):

$$D_U(U_2)=\vartheta$$

Our simplified model assumes a fixed endurance threshold (θ); however, in real-world applications, θ is unlikely to be static. Instead, it is more plausibly a dynamic value

influenced by a range of contextual factors such as national morale, inflation rates, access to global trade, and political stability.

To enhance the model's realism and predictive power, θ could be reconceptualised as a time-dependent function: θ(t) = f(political stability, external support, morale). This would allow for a more adaptive threshold that reflects the fluidity of wartime resilience. Moreover, θ could be empirically estimated by examining historical cases of systemic exhaustion under prolonged attritional pressure, such as the U.S. withdrawal from Vietnam or the fall of the Afghan government in 2021. Such calibrations would ground the model in real-world precedents and strengthen its applicability to current or future conflicts.

The definitions of E and ϑ are detailed as follows:

Symbol	Meaning	Type	Interpretation
E	Equilibrium threshold (in time)	Temporal variable (e.g., quarter index)	The last point in time at which the war economy is still sustainable - that is, when P(Q)≥C(Q). Beyond E, the system enters unsustainability mode.
θ	Endurance threshold (in magnitude)	Scalar value (e.g., unit count, resource capacity)	The maximum cumulative deficit the system can absorb before collapse. Once the integral D_U exceeds θ, the system collapses at time U_2.

Under the assumption that industrial production (i.e. supply) cannot fully compensate for wartime consumption (i.e. demand), we posit that the duration of conflict (started at the baseline point *B*) leads to a decline in the production-to-consumption ratio (P/C). Following that trend, it reaches the Equilibrium threshold, denoted as point *E*, which represents the last moment at which the country is able to maintain its war efforts (military operations) with production continuing to match or exceed consumption. Up to this point, the war economy has operated in a "sustained mode", meaning the system remains in a dynamic equilibrium. Beyond point *E*, the system, therefore, transitions into an "unsustained mode", where military demand exceeds production capacity, leading to a depletion of resources.

Net Sustainment Capacity represents the war economy function, where P(t) is the rate of military production and C(t) is the rate of consumption, at time *t* (also expressed in quarters Q).

$$w_e(t) = P(t) - C(t) \quad (1)$$

This equation would be different in "real circumstances", comprising also foreign aid (for example, from North Korea) as A(t). However, for the sake of simplicity and due to the limited data available, we intentionally omitted that variable.

$$w_e(t) = P(t) + A(t) - C(t)$$

The cumulative sustainment capacity or total net production effort during the sustainable phase of the war economy function, over a time interval $[B, E]$, can then be represented as:

$$D_s = \int_B^E w_e(t)\, dt \qquad (2)$$

If $w_e(t) < 0$ for extended periods (i.e., consumption and losses consistently exceed production), the system enters an unsustainable regime, quantified by the cumulative deficit D_U, progressing toward systemic failure (i.e. collapse) at the time U_2.

When the war economy function reaches the point U_1, defined as $E + 1$, it enters an unsustainable phase in which the ratio $P(t)/C(t) < 1$ holds persistently, indicating that losses (demand) outpace replenishment (supply). During this phase, the war economy function remains negative, and cumulative unsustainability over time can be modelled as:

$$D_U = \int_{U_1}^{U_2} w_e(t)\, dt \qquad (3)$$

Where:

- D_U is the total unsustainable war effort (the accumulated demand or resource pressure after the critical threshold "E".
- U_2 denotes the point in time at which the accumulated deficit exceeds the system's endurance threshold, rendering continued conflict unsustainable.

The use of integral calculus in this model is not merely symbolic but vital for capturing the temporal nature of attritional warfare. Unlike discrete battles or supply events, attritional dynamics develop continuously over time, with losses and replenishment gradually adding up. By expressing production and consumption (losses) as functions of time, and integrating over a specified interval, we can model the total net sustainment of the military system. This method reflects the true reality of modern warfare: states do not lose or gain military capacity in isolated instances, but through a constant flow of material and human resources, driven by the rhythm of combat and logistics.

Once the unsustainability integral D_U reaches the system's endurance threshold at time U_2 the state cannot expect to continue the war efforts effectively[2], because the war economy is no longer functioning adequately to provide the needed resources. As

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2   It has to be emphasised that this critical threshold is qualitative and context-dependent (i.e., system-specific, and not numerically fixed)

a result, the warring side that reaches this state is poised to lose in the war of attrition because the burden is too heavy to bear.

Our second objective **(O$_2$)** is to determine when the country will reach the system's endurance threshold at time , and what factors affect the speed of change. To achieve this, we analyse the elasticity of the function, as shown in Figure 1.

We define sustained war economy (D_s) of the function $f(w_e)$ as:

$$P/C > 1 \quad (4)$$

and unsustained war economy D$_U$ of the function $f(w_e)$ as:

$$P/C < 1 \quad (5)$$

We concluded that the war economy function $f(w_e)$ is highly elastic and sensitive to changes in the production-to-consumption ratio (P/C), as illustrated in Figure 1, which shows the case of the consistent decline in sustainment capacity over time.

Building on this theoretical foundation, Figure 2 illustrates the velocity of the War Economy Function, defined as the rate of change in Net Sustainment Capacity (NSC). Negative velocity indicates that consumption (losses) persistently exceeds production (replenishment), resulting in a continuous decline in NSC and pushing the system toward the critical failure threshold at time U_2.

When the P/C ratio falls, the NSC declines. With the estimated logarithmic trend, the rate of decline diminishes over time. Mathematically, the persistence of unsustainability is captured by the velocity of the war economy function,

$$v(Q) = \frac{d}{dQ}[P(Q) - C(Q)] = P'(Q) - C'(Q) \qquad (6)$$

Empirically, using quarterly data, we approximate *v* by the first difference ΔNSC(Q) = NSC(Q) − NSC(Q−1). A persistently negative velocity, therefore, reflects ongoing unsustainability (i.e. losses exceed production), even as the rate of decline may diminish with time.
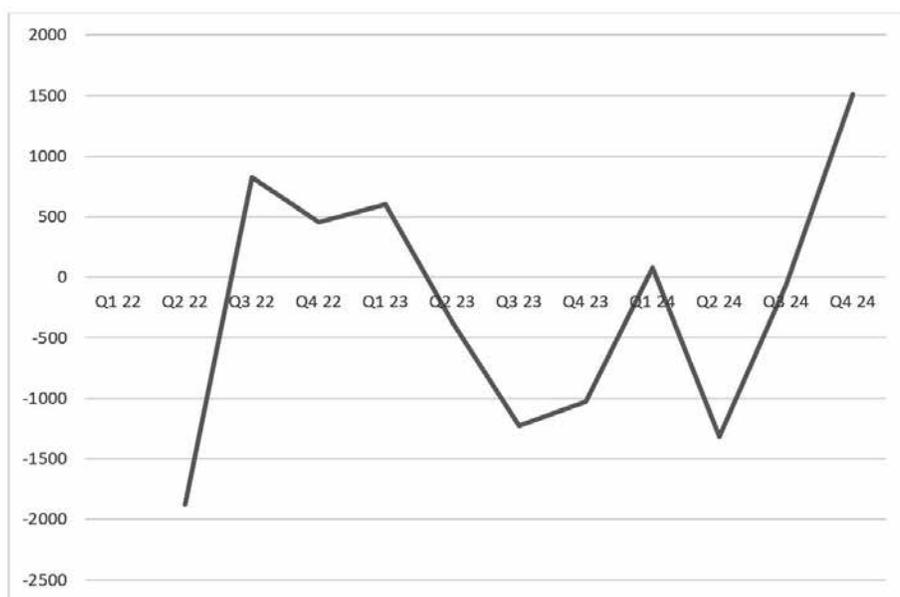
Figure 2: The velocity of Russia's Net Sustainment Capacity
Source: Data from Table 9

Note: Figure 2 illustrates the velocity of Net Sustainment Capacity (ΔNSC), defined as the quarter-to-quarter change in the balance between Russian military production and battlefield losses. The x-axis represents quarters from Q1 2022 to Q4 2024 (timeline of the war), and the y-axis shows the velocity of Net Sustainment Capacity (ΔNSC), i.e., the change in the production–loss gap from the previous quarter. Negative values indicate that the gap widened, while positive values indicate that it narrowed. Unlike the earlier theoretical illustration, this figure is derived directly from the empirical data presented in the Appendix tables.

Negative values in Figure 2 indicate periods where losses increased faster than production, accelerating the erosion of Russia's war economy. Positive values reflect temporary slowdowns in this decline, usually due to increased production or reduced losses. Although fluctuations are visible, the overall pattern confirms persistent instability: NSC remains negative throughout the period, and velocity repeatedly dips below zero. This demonstrates that the Russian war economy is not only unsustainable at the level (Figure 1) but also subject to volatile and often accelerating decline, underscoring its structural vulnerability in a prolonged attritional conflict.

We conclude that as the production-to-consumption ratio (P/C) decreases, the burden of attrition increases, forcing the system closer to the critical threshold . Mathematically,

once the function has entered the unsustainable regime , velocity (as defined in equation 6) remains negative, indicating persistent decline, although its magnitude diminishes over time. The system ultimately converges toward the endurance threshold at U_2:

$$\lim_{Q \to U_2} f(w_e)(Q) = \theta \qquad (7)$$

i.e. as the war economy function approaches the critical point , it reaches the endurance threshold θ. This trend would only reverse if consumption (losses) decreases, or if overall production exceeds consumption.

Once the system has entered the unsustainable regime *(D_U)*, the condition

$$v(Q)<0 \Leftrightarrow P'(Q)<C'(Q) \quad (8)$$

holds, meaning that the marginal growth of losses outpaces the marginal growth of production. This is equivalent to the inequality:

$$\frac{dP}{dQ} < \frac{dC}{dQ} \qquad (9)$$

A reversal of the trend would require non-negative velocity, i.e.

$$v(Q) \geq 0 \Leftrightarrow \frac{dP}{dQ} \geq \frac{dC}{dQ} \qquad (10)$$

while a reversal of the cumulative deficit would require production to at least match consumption:

$$C(Q)-P(Q) \leq 0 \Leftrightarrow P(Q) \geq C(Q) \quad (11)$$

Under the estimated logarithmic trends, however, these conditions are never met:

$$P'(Q) - C'(Q) = Q - 1218.44 < 0,$$

$$P(Q) - C(Q) = (-1218.44)\ln Q - 793.84 < 0 \; (\forall Q \geq 1) \qquad (12)$$

Where ∀Q means "for all" or "for every"**.** It reads: P(Q)–C(Q) is negative for all quarters Q greater than or equal to 1.

Thus, within the observed period, both the slope of the Net Sustainment Capacity and its level remain negative. The Russian war economy is therefore locked into persistent unsustainability unless a structural change reduces consumption or significantly increases production.

2.2   Empirical Analysis of the Sustainability of the War Economy

Building on the theoretical framework, our third objective (O_3) is to utilise empirical data to assess the current state of the Russian war economy. We drew loss data from Minfin[3] (Minfin, 2024) and matched it with production estimates for the same categories. Where official production figures were unavailable, we approximated values using Cooper (2024) and Cavoli (2024).

This data is shown in tables A1-A3. Next, we considered the production of the same three parameters that the Russian military-industrial complex produces. These are shown in Tables A4-A7. We then compiled the losses according to all parameters into a single function, referred to as the "function of production" (f1). Following this step, the same three parameters and their respective data showing losses are used to create a "function of consumption" (f2). Combining the movements from both functions, based on the data in Table A7, we designed a graphical representation (Figure 3) of the regression-based trendlines for Russian military production and losses.
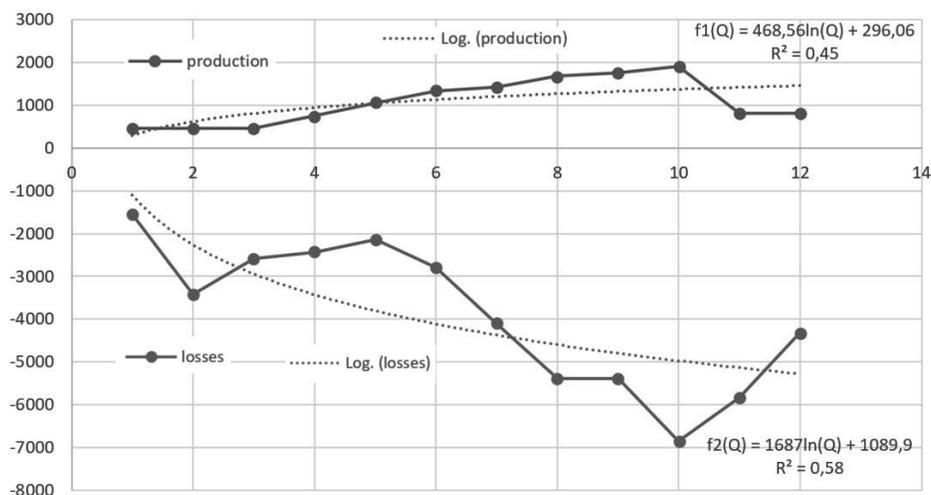
Figure 3: Regression-based trendlines of Russian military production and losses between 2022 and 2024
Source: Logarithmic regression on data in Table A8

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3   Minfin.com.ua is a private financial portal that provides information on currency exchange rates, banking services, investment options, and financial news in Ukraine. While it offers valuable financial data and tools, it is not affiliated with the Ukrainian government.

Note: The figure displays logarithmic regression trendlines for Russian military production (positive values) and battlefield losses (plotted as negative values) from Q1 2022 to Q4 2024. The x-axis represents quarterly intervals, expressed as the natural logarithm of time (ln(Q)), and the y-axis represents the number of military platforms (aggregated tanks, armoured vehicles, and artillery). The figure shows that production values consistently lag behind consumption values, indicating that the Russian war economy remains structurally unsustainable throughout the observed period.

For modelling purposes, we defined the baseline (Q4 2021) as the point of equilibrium, where production and consumption are assumed to be equal, while amortisation effects are held negligible (*ceteris paribus*). In reality, pre-conflict consumption was negligible, but this assumption provides a neutral reference point for the sustainment function. These baseline conditions yielded the initial equation for war economy sustainment *(D_s)*.

$$f_1(Q) = f_2(Q) = 1 = D_s \quad (13)$$

From point (D_s), the function $f_1(Q)$ can be assumed to follow the trend line approximated by the regression analysis:

$$f_1(Q) = 468.56 \ln(Q) + 296.06, \text{ and thus } \lim_{Q \to \infty} f_1(Q) = +\infty \quad (14)$$

Equation (14) defines the regression function for production. Its limit as $Q \to \infty$ is unbounded, meaning that if the war continued indefinitely, production would rise without bound under this trend.

The second function *$f_2(Q)$,* can also be assumed to follow the trend line achieved through the regression analysis as:

$$f_2(Q) = 1687.00 \ln(Q) + 1089.90, \text{ and thus } \lim_{Q \to \infty} f_2(Q) = +\infty \quad (15)$$

Taken together, these regressions imply that in every quarter of the observed period (Q1 2022–Q4 2024), production remains below consumption. Formally,

$$NSC(Q) = P(Q) - C(Q) = (-1218.44)\ln Q - 793.84 < 0 \ (\forall Q \geq 1).$$

This means that *P(Q) < C(Q)* throughout the sample, and the equilibrium point *E*, where production and consumption would be equal, lies outside the observed window.

Both functions reasonably follow their estimated trend lines, with coefficients of determination (R^2) indicating a moderate statistical fit. Equations (14) and (15) reveal

a widening gap between production and consumption, with estimated losses consistently exceeding output. This empirical pattern supports the conclusion that the Russian war economy function (w_e) is unsustainable (D_U) and is moving toward the critical breaking threshold (U_2).

The regression analysis yields the following trendlines for Russian military production and losses over time (Q1 2022–Q4 2024):

$$P(Q) = 468.56 \ln(Q) + 296.06 \ (R^2 = 0.45, SEE = 413.80)$$

$$C(Q) = 1687 \ln(Q) + 1089.9 \ (R^2 = 0.58, SEE = 1146.10)$$

Where: R^2 = the coefficient of determination, and SEE= Standard Error of the Estimate

To construct the dataset for the regression analysis, we organised the timeline of the conflict into twelve sequential quarters (Q1 2022–Q4 2024), assigning each quarter an integer index (Q = 1…12) to serve as the temporal variable. We then applied the natural logarithm transformation to Q to capture the non-linear trajectory of production and consumption over time, consistent with the logarithmic regression model employed. Production and consumption (losses) values for each quarter were extracted from Table A7, which aggregates Russian military output and losses across three key categories: battle tanks, armoured vehicles, and artillery systems. These data were compiled into a structured dataset (Table A8), which contains the quarter label, the logarithmic time variable ln(Q), and the corresponding production and consumption figures. This dataset formed the empirical basis for estimating the regression equations and interpreting the sustainability of the Russian war economy under conditions of prolonged attritional warfare.

The results indicate that the consumption (losses) function exhibits a somewhat stronger fit (R^2) than the production function, although both models reveal clear directional trends. The relatively high standard errors point to volatility in production and consumption values, underscoring the need for cautious interpretation in strategic planning. These findings also confirm that the Russian war economy is structurally unable to supply its military with the required resources (tanks, armoured vehicles, and artillery), as quarterly losses consistently exceed quarterly production.

3   DISCUSSION

The findings indicate that the Russian war economy is structurally unsustainable, as military production consistently lags behind battlefield losses. The regression trends demonstrate a continued progression toward the endurance threshold, reflecting the inability of internal production to compensate for rising attrition rates.

We acknowledge that the empirical data used in this study are approximate and, in many cases, classified. Moreover, the aggregation of platforms such as tanks, artillery, and armoured vehicles is not a fully adequate way to capture the complexity of warfare. Nevertheless, our intention is to apply defence economics, with its empirical aspects, to provide insights that can support strategic planning. In this regard, the value of our approach lies less in the precision of the figures than in the modelling framework itself, which offers a useful tool for strategic planning under conditions of uncertainty.

This unsustainability is not merely a logistical issue but a strategic vulnerability. It supports the hypothesis that modern attritional warfare may exhaust a nation's warfighting capacity regardless of tactical performance. The function's sensitivity to production-consumption ratios confirms the high elasticity of war economies in prolonged conflicts.

These results also highlight the necessity of external support and strategic prioritisation of sustainment over doctrinal preferences. In the case of Ukraine, for example, attritional endurance may present an opportunity rather than a constraint, depending on external aid and internal resolve.

However, the model presented in this research rests on the assumption of a closed economic system with internally driven production and losses. This assumption is a necessary simplification for theoretical clarity, but does not fully account for significant exogenous variables, notably, foreign military support. For instance, extensive North Korean munitions deliveries to Russia, documented since mid-2023, have supplied an estimated 4.2 to 5.8 million artillery rounds and rockets across 64 shipments, likely contributing to 750 monthly containers (Armenzoni et al., 2025). These shipments offset internal production constraints, enabling sustained Russian artillery use on the battlefield. This reality introduces a substantial deviation from the idealised function used in this paper.

As such, while the internal unsustainability of Russia's war economy is evident, external support can delay or even temporarily obscure systemic economic failure. This reinforces the importance of consistent and scaled Western aid to Ukraine in maintaining military equilibrium and countering Russia's quantitative advantage gained through alliances with states like North Korea. Thus, while the model remains valid as a framework for endogenous sustainability, its application in real-world contexts must be complemented by political-economic intelligence on foreign resupply dynamics.

The theoretical model developed here quantifies operational tipping points and offers a replicable approach to evaluating other war economies facing prolonged resource depletion. The relevance of any similar model depends on the context and the dynamics of war, whose nature and evolving character, with its inherent uncertainty, is widely discussed in military theory, particularly by Carl von Clausewitz (1984).

Another conceptual parallel can be drawn between Clausewitz's notion of the "culminating point" and the "endurance threshold" (θ) developed in our model. While originating from different domains (Clausewitz's from strategic theory and ours from economic

modelling), both concepts describe the critical juncture at which continued offensive action becomes unsustainable. The comparison below outlines the similarities and differences between these two thresholds in terms of definition, nature, underlying mechanisms, and outcomes.

Comparison of Clausewitz's "Culminating Point" and the authors' "Endurance Threshold (θ) may be presented as:

Concept	Clausewitz's "Culminating Point"	Authors' Model "Endurance Threshold" (θ)
Definition	The point at which the strength of an advancing force declines to the point that it can no longer achieve a decisive victory and may risk defeat.	The point at which cumulative losses exceed a state's capacity to replace them, rendering further war effort unsustainable.
Nature	Strategic / Operational	Economic / Quantitative
Mechanism	Caused by overextension, logistical limits, friction, or moral decay.	Caused by the imbalance between military losses and production.
Effect	Offensive turns into vulnerability; it requires consolidation or retreat.	Military capacity collapses; continued operations become non-viable.
Result	Strategic failure is not recognised or mitigated.	Systemic exhaustion and likely defeat in a war of attrition.

The endurance threshold (θ) functions analogously to Clausewitz's culminating point, but with clearer economic measurability. In bargaining models, this point may correlate with an increased willingness to negotiate, especially if actors perceive continued fighting as unsustainable. Thus, economic modelling of θ can inform when a state might shift from maximalist demands to settlement-seeking behaviour.

Attritional warfare shifts the nature of strategic signalling. While high losses may project commitment, a prolonged inability to replenish resources can weaken credibility. In our model, once the P/C ratio declines persistently, continued combat may signal desperation rather than resolve. This reversal could invite adversaries to escalate offensives or delay negotiations, anticipating further decay.

CONCLUSION

This research establishes a comprehensive theoretical and empirical framework to analyse the dynamics of war economies during attritional warfare. Applying the attritional warfare model to the Russian-Ukrainian war highlights critical points of unsustainability

within the Russian war economy, characterised by a growing disparity between military production and consumption. Our model, considering available data and related trends, indicates that Russia's war of attrition is currently (at the end of 2024) unsustainable, approaching the endurance threshold. In the military context, that would characterise the state where the combat power is limited to the point where it cannot achieve military and political goals.

The regression analysis reveals a moderate correlation between production trends and consumption trends. The results indicate that Russian military production cannot keep pace with escalating losses, confirming the unsustainability of the war economy. It, however, remains relevant under specific circumstances, which include the continuing will to fight on the defending side (in this case, Ukraine), continuing and sustained production of military capabilities (including platforms, armament, ammunition, equipment, training and maintenance, among others) or aid from outside (which is currently the case with the Western support to Ukraine). Generally, the model and analysis presented in this research require the existence of specific and measurable trends to be relevant for supporting strategic decision-making. Consequently, the main contribution of the research to defence economics is its methodology.

By focusing on the rate of change in Net Sustainment Capacity (NSC), this analysis provides a practical tool for intelligence and strategic forecasting. Monitoring how quickly production falls behind losses allows analysts to identify early warning signs of systemic strain and to estimate when, in this case, Russia may approach its endurance threshold (θ). The persistent negative trend observed in the model underscores the structural unsustainability of the Russian war economy, where losses consistently exceed production, even if foreign aid can temporarily delay collapse. Translating these economic dynamics into measurable indicators enables policymakers to better calibrate support, anticipate negotiation windows, and exploit adversary vulnerabilities in a prolonged war of attrition.

From a policy standpoint, economic sustainment models can inform the timing and scale of aid. In the case of Ukraine, support aimed at prolonging Russia's path to the θ threshold could induce earlier Russian willingness to negotiate. Conversely, misestimating the threshold risks prolongs a costly conflict without strategic gain. Future research could integrate the endurance threshold into dynamic models of conflict bargaining to better predict negotiation windows. Additionally, empirical calibration of θ across past conflicts could offer new tools for forecasting conflict resolution timing.

Key contributions include (1) identifying critical thresholds of economic sustainability in war, (2) emphasising the importance of internal resource allocation and external support to sustain prolonged engagements, and (3) offering a methodological approach to assess war economies in similar conflicts. This makes the research a valuable tool for understanding the longevity and limitations of attritional warfare, offering insights that extend beyond the specific case of the Russian-Ukrainian conflict. Future applications of this model could assist policymakers and military strategists in forecasting and

addressing the economic challenges of prolonged armed conflicts and provide attritional warfare as a relevant strategic choice.

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Appendix A: Data source

Table A1: Battle-tank losses by the Russians between 2022 and 2024

Year	Quarter	No. of lost platforms
2022	Q1	414
	Q2	952
	Q3	777
	Q4	677
2023	Q1	585
	Q2	428
	Q3	661
	Q4	1281
2024	Q1	1003
	Q2	1113
	Q3	784
	Q4	566

Source: According to available data (Armed Forces of Ukraine 2024)

Table A2: Russia's armoured-vehicle losses between 2022 and 2024

Year	Quarter	No. of lost platforms
2022	Q1	889
	Q2	1985
	Q3	1213

	Q4	1135
2023	Q1	897
	Q2	887
	Q3	1123
	Q4	2096
2024	Q1	2234
	Q2	2226
	Q3	787
	Q4	1776

Source: Aaccording to available data (Armed Forces of Ukraine 2024)

Table A3: Russia's artillery system losses between 2022 and 2024

Year	Quarter	No. of lost platforms
2022	Q1	231
	Q2	480
	Q3	601
	Q4	624
2023	Q1	662
	Q2	1479
	Q3	2315
	Q4	2005
2024	Q1	2601
	Q2	3517
	Q3	4256
	Q4	1975

Source: According to available data (Armed Forces of Ukraine 2024)

Table A4: Russian production of battle-tanks between 2022 and 2024

Year	Quarter	No. of produced platforms
2022	Q1	*175
	Q2	*175
	Q3	*175
	Q4	123
2023	Q1	186
	Q2	243
	Q3	270
	Q4	328
2024	Q1	360
	Q2	387
	Q3	**300
	Q4	**300

Source: According to available data (Wolff et al. 2024). Approximated values are marked with an asterisk (*).

Table A5: Russian production of armoured vehicles between 2022 and 2024

Year	Quarter	No. of produced platforms
2022	Q1	*236
	Q2	*236
	Q3	*236
	Q4	585
2023	Q1	814
	Q2	1020
	Q3	1069
	Q4	1251
2024	Q1	1290

	Q2	1409
	Q3	479
	Q4	*479

Source: According to available data (Wolff et al. 2024). Approximate values are marked with an asterisk (*).

Table A6: Russian production of artillery platforms between 2022 and 2024

Year	Quarter	No. of produced platforms
2022	Q1	*45
	Q2	*45
	Q3	*45
	Q4	45
2023	Q1	63
	Q2	79
	Q3	82
	Q4	98
2024	Q1	102
	Q2	112
	Q3	38
	Q4	*38

Source: According to available data (Wolff et al. 2024). Approximate values are marked with an asterisk (*)

Table A7: Combined Russia's production and consumption of tanks, armoured vehicles and artillery

Year	Quarter	Production	Consumption
2022	Q1	456	1543
	Q2	456	3417
	Q3	456	2591
	Q4	753	2436

2023	Q1	1063	2144
	Q2	1342	2794
	Q3	1421	4099
	Q4	1677	5382
2024	Q1	1752	5382
	Q2	1908	6856
	Q3	817	5827
	Q4	817	4317

Source: Aggregated data from tables A1 to A6

Note: *In the empirical model, consumption (C) denotes the magnitude of losses and is treated as positive (tables report raw losses as positive counts).*

Table A8. Quarterly Russian military production and consumption with logarithmic time index (Q1 2022 – Q4 2024).

Quarter	ln(Q)	Production	Consumption
Q1 2022	0.000000	456	1543
Q2 2022	0.693147	456	3417
Q3 2022	1.098612	456	2591
Q4 2022	1.386294	753	2436
Q1 2023	1.609438	1063	2144
Q2 2023	1.791759	1342	2794
Q3 2023	1.945910	1421	4099
Q4 2023	2.079442	1677	5382
Q1 2024	2.197225	1752	5382
Q2 2024	2.302585	1908	6856
Q3 2024	2.397895	817	5827
Q4 2024	2.484907	817	4317

Source: Aggregated data from Table A7

Table A9. Quarterly Russian Military Production, Losses (Consumption), Net Sustainment Capacity (ΔNSC), and Velocity (2022–2024)

Quarter	Production	Consumption	NSC	Velocity (ΔNSC)
Q1 2022	456	1543	-1087	
Q2 2022	456	3417	-2961	-1874
Q3 2022	456	2591	-2135	826
Q4 2022	753	2436	-1683	452
Q1 2023	1063	2144	-1081	602
Q2 2023	1342	2794	-1452	-371
Q3 2023	1421	4099	-2678	-1226
Q4 2023	1677	5382	-3705	-1027
Q1 2024	1752	5382	-3630	75
Q2 2024	1908	6856	-4948	-1318
Q3 2024	817	5827	-5010	-62
Q4 2024	817	4317	-3500	1510

Source: According to data derived from Table A8